Integration 1

1. Evaluate $\int_{-1}^{1} \frac{1+x^{2016}}{1+2^x} dx$.

2. **(a)** Consider the integrals: $I_1 = \int \frac{\cos x}{\cos x + \sin x} dx$ and $I_2 = \int \frac{\sin x}{\cos x + \sin x} dx$

By evaluating $I_1 + I_2$ and $I_1 - I_2$, find the values of I_1 and I_2 .

(b) Evaluate $\int \frac{\cos x}{a \cos x + b \sin x} dx$, where $a^2 + b^2 \neq 0$.

(c) Evaluate $\int \frac{e^x}{a e^x + b e^{-x}} dx$, where $a, b \neq 0$.

3. **(a)** By using the substitution $x = \alpha \cos^2 \theta + \beta \sin^2 \theta$, evaluate the integral

$\int_{\alpha}^{\beta} \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx$, where $\alpha < \beta$.

Evaluate this integral where $\alpha > \beta$.

(b) By using the substitution $t = \frac{1}{x}$, evaluate the integral

$\int_{a}^{b} \frac{1}{t\sqrt{(t-a)(b-t)}} dt$, where $0 < a < b$.